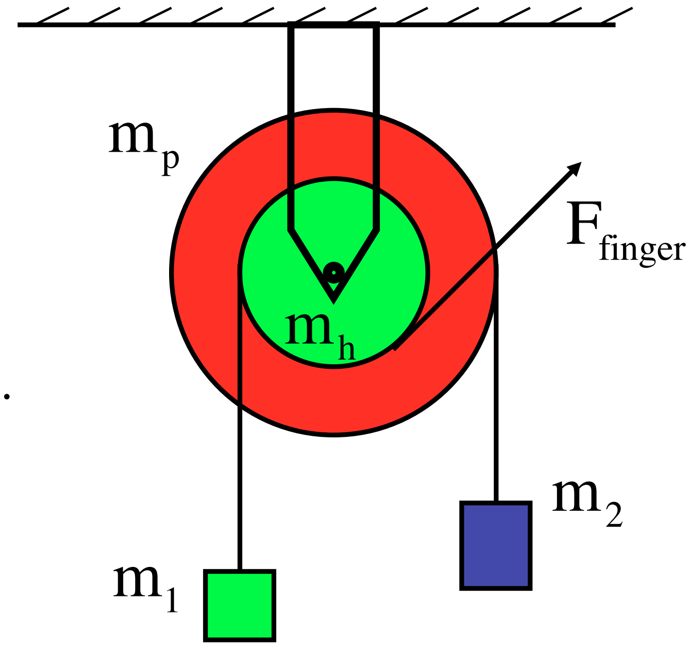
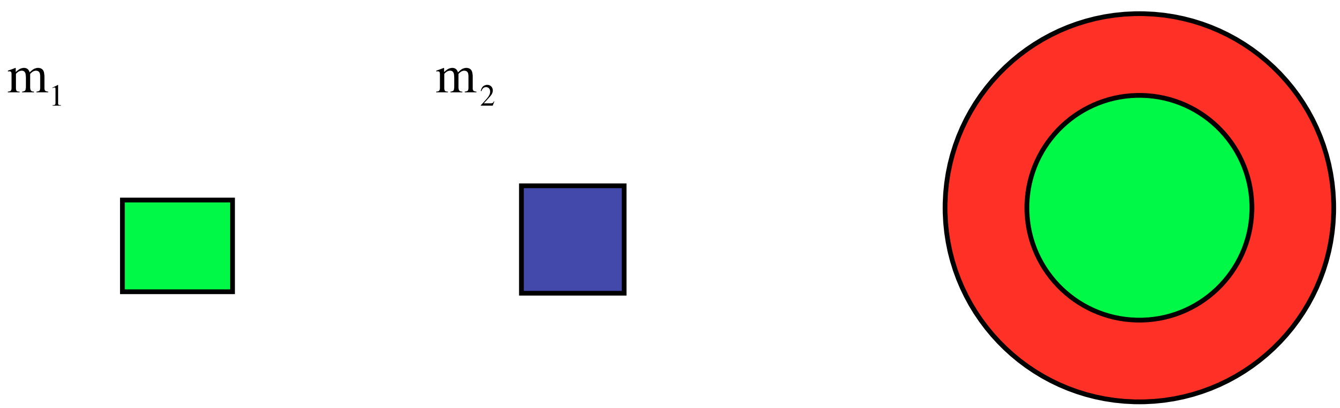
7.) A massive hub of radius 2R/3 is glued to a massive pulley of radius R. Your finger acting tangent to the edge of the hub holds the system stationary. Strings are wound around both the hub and pulley with masses attached to each (see sketch). What is known is:

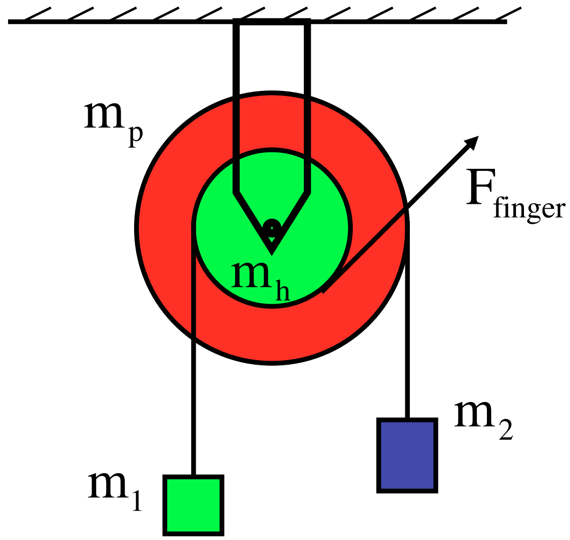
m_1, m_2, m_p, m_h, R, g, and for each disk $I_{cm} = \frac{1}{2}mr^2$.



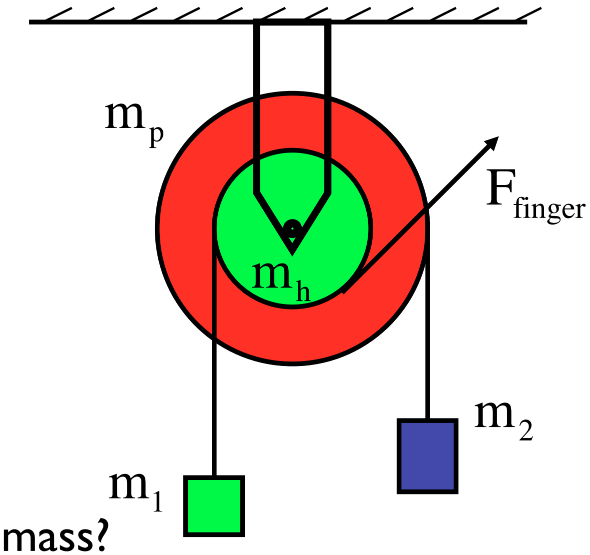
a.) Draw a f.b.d. for both masses and the pulley/hub assembly.



b.) How much force must your finger apply to keep the system stationary?



c.) What is the moment of inertia of the pulley/hub complex.



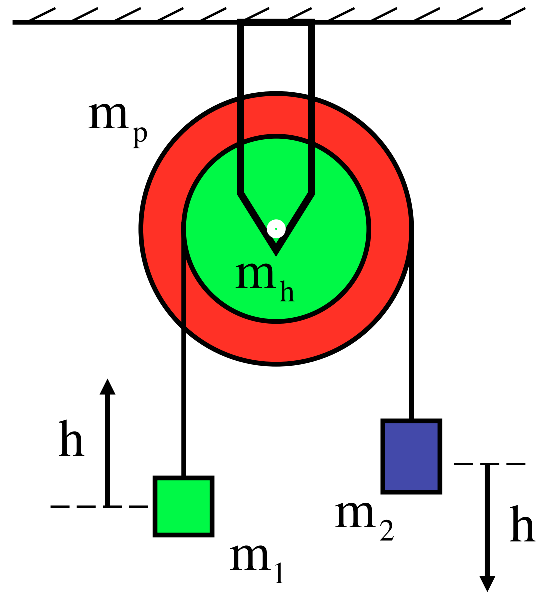
d.) You remove your finger and the system begins to accelerate. What is the *angular acceleration* of the pulley/hub?

e.) What is the magnitude of the acceleration of the two hanging mass?

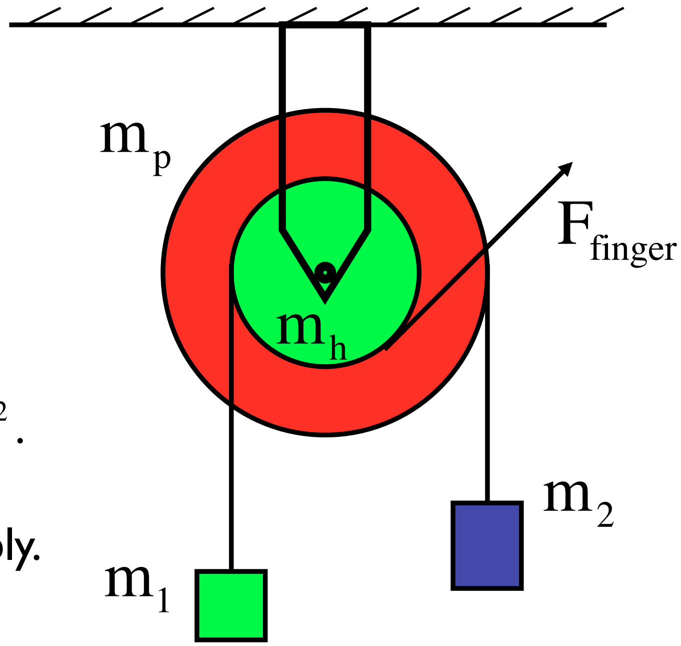
f.) The right hanging mass freefalls a distance "h." What is the pulley's *angular velocity* at the end?

g.) What is are the velocity magnitudes for the hanging masses for the situation in #e?

h.) What is the hub's *angular momentum* L for the situation in Part e?

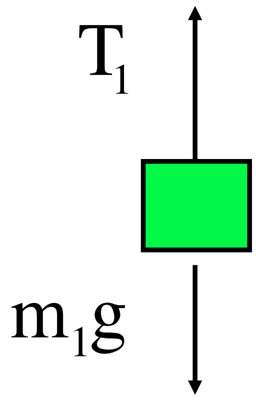


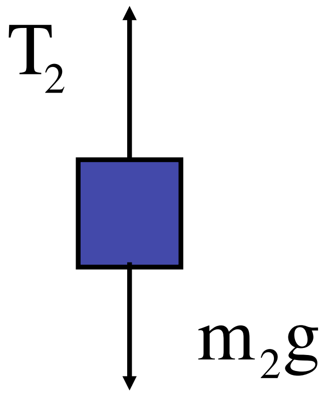
7.) A massive hub of radius 2R/3 is glued to a massive pulley of radius R. Your finger acting tangent to the edge of the hub holds the system stationary. Strings are wound around both the hub and pulley with masses attached to each (see sketch). What is known is:

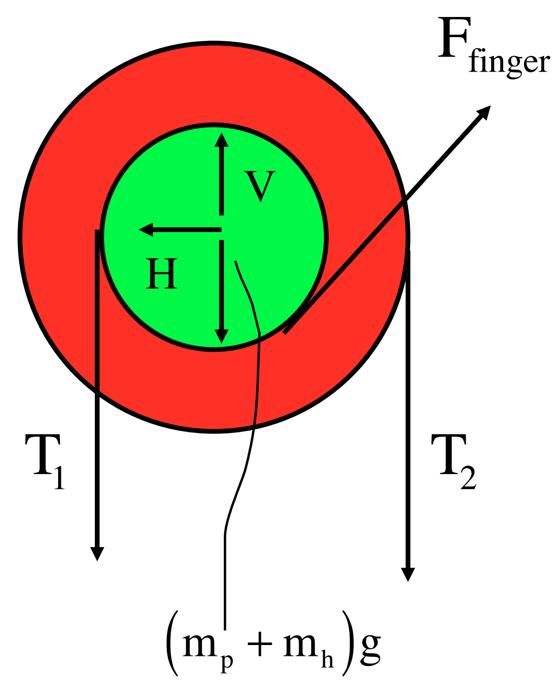


$$m_1,\ m_2,\ m_p,\ m_h,\ R,\ g,\ \text{and for each disk } I_{cm} = \frac{1}{2}mr^2.$$

a.) Draw a f.b.d. for both masses and the pulley/hub assembly.



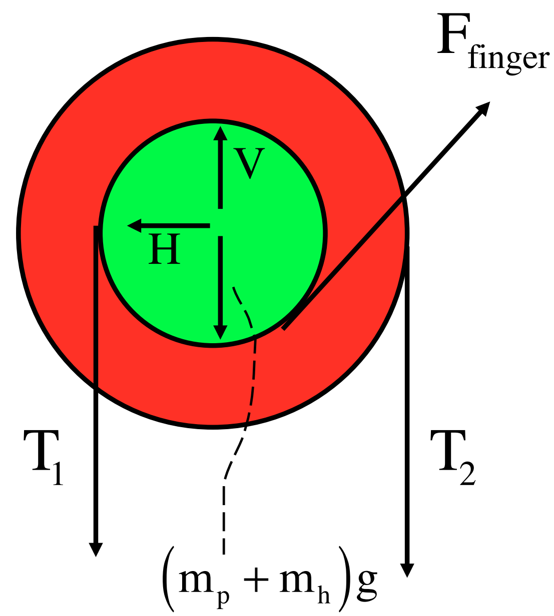




b.) How much force must your finger apply to keep the system stationary?

This is a "torque about the pin" problem.

Note: Because *nothing is accelerating*, the tensions are just equal to the weights of the hanging masses.



$\sum \Gamma_{pin}$:

$$\cancelto{0}{\Gamma_H} + \cancelto{0}{\Gamma_V} + \cancelto{0}{\Gamma_{Mg}} + (T_1)\left(\frac{2}{3}R\right) - (T_2)R + F_{finger}\left(\frac{2}{3}R\right) = \cancelto{0}{I_{pin}\alpha}$$

$$\Rightarrow \quad (m_1 g)\left(\frac{2}{3}R\right) - (m_2 g)R + F_{finger}\left(\frac{2}{3}R\right) = 0$$

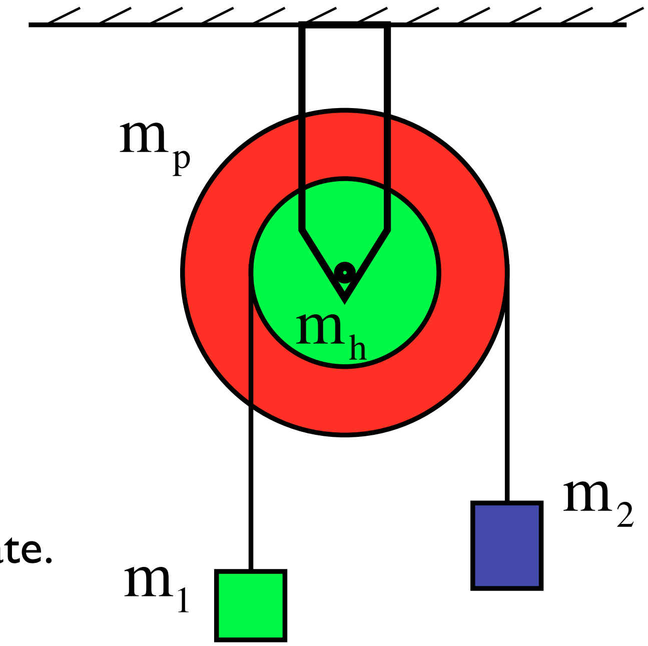
$$\Rightarrow \quad F_{finger} = \left(-m_1 + \frac{3}{2}m_2\right)g$$

c.) What is the *moment of inertia* of the pulley/hub complex.

$$I_{total} = I_{hub} + I_{pulley}$$

$$= \frac{1}{2}m_h\left(\frac{2R}{3}\right)^2 + \frac{1}{2}m_pR^2$$

$$= \left(\frac{2m_h}{9} + \frac{m_p}{2}\right)R^2$$



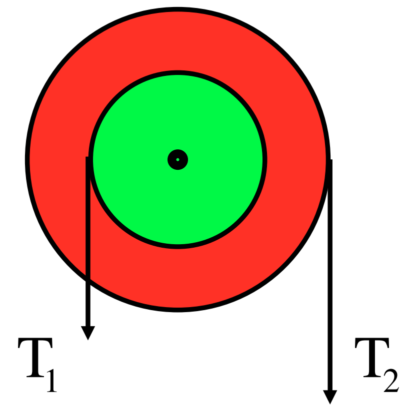
d.) You remove your finger and the system begins to accelerate. What is the *angular acceleration* of the pulley/hub?

$$\sum \Gamma_{pin}:$$

$$T_1\left(\frac{2}{3}R\right) - T_2R = I_{pin}\alpha$$

$$T_1\left(\frac{2}{3}\cancel{R}\right) - T_2\cancel{R} = \left(\left(\frac{2m_h}{9} + \frac{m_p}{2}\right)R^{\cancel{2}}\right)\alpha$$

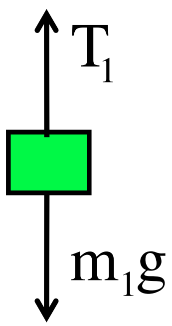
$$\Rightarrow \quad \left(\frac{2}{3}\right)T_1 - T_2 = \left(\left(\frac{2m_h}{9} + \frac{m_p}{2}\right)R\right)\alpha$$



Three unknowns--we need two more equations!

Whereas the *angular acceleration* of both the pulley and the hub are the same, the *acceleration* of a point on the hub's edge (and, by extension, the string wound over the hub) will be different than the acceleration of a point on the pulley's edge. That means N.S.L. will yield expressions for each with *different* acceleration terms. N.S.L. yields:

for m_1

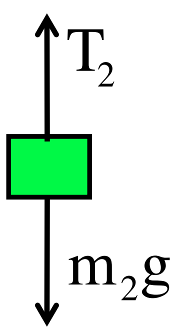


$$\sum F_y:$$

$$T_1 - m_1 g = -m_1 a_1$$

$$\Rightarrow \quad T_1 = m_1 g - m_1\left(\frac{2}{3}R\alpha\right) \quad \text{as} \quad a_1 = \left(\frac{2}{3}R\right)\alpha$$

for m_2



$$\sum F_y:$$

$$T_2 - m_2 g = m_2 a_2$$

$$\Rightarrow \quad T_2 = m_2 g + m_2 (R\alpha) \quad \text{as} \quad a_2 = R\alpha$$

Substituting the tension expressions into, then simplified the "sum or torques" expression yields:

$$\left(\frac{2}{3}\right) T_1 - T_2 = \left(\frac{2m_h}{9} + \frac{m_p}{2}\right) R\alpha$$

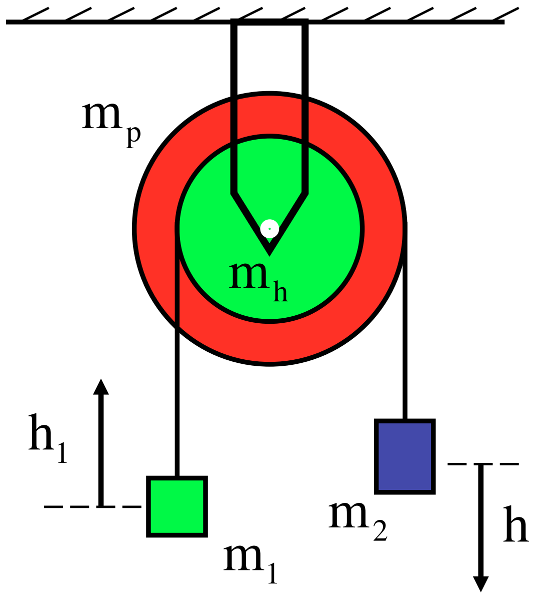
$$\left(\frac{2}{3}\right)\left(m_1 g - m_1\left(\frac{2}{3}R\alpha\right)\right) - \left(m_2 g + m_2 (R\alpha)\right) = \left(\frac{2m_h}{9} + \frac{m_p}{2}\right) R\alpha$$

$$\Rightarrow \quad \frac{2}{3} m_1 g - \frac{4}{9} m_1 R\alpha - m_2 g - m_2 (R\alpha) = \left(\frac{2m_h}{9} + \frac{m_p}{2}\right) R\alpha$$

$$\Rightarrow \quad \alpha = \frac{\frac{2}{3} m_1 g - m_2 g}{\left(\frac{2m_h}{9} + \frac{m_p}{2}\right) R + m_2 R + \frac{4}{9} m_1 R}$$

e.) What is the magnitude of the acceleration of the two hanging mass?

$$a_2 = R\alpha \qquad a_1 = \left(\frac{2}{3}R\right)\alpha$$



f.) The right hanging mass freefalls a distance "h." What is the pulley's *angular velocity* at the end? (Sketch on next page.)

This is a *conservation of energy* problem. The difficulty is that because the strings are each accelerating at a different rate (remember, the hub and pulley angular accelerations and angular velocities are the same but their edge accelerations and velocity are not), the string (and hence mass) velocity magnitudes will not be the same and neither will be the distances traveled. Going through a similar analysis as was done with the accelerations, we can write:

$$v_1 = \left(\frac{2}{3}R\right)\omega \quad \text{and} \quad v_2 = R\omega$$

An additional consequence is the drop distances will be different. Look at the pulley. One rotation moves the string one circumference worth, or $2\pi R$. During that same motion, the hub will only feed out two-thirds worth of string, or $2\pi(2R/3)$. In other words, the displacement of m_1 will be 2/3 the displacement of m_2, or

$$h_1 = \frac{2}{3}h.$$

f.) With all of that in mind, and assuming the pulley's mass falls a distance "h" with the *zero potential energy levels* placed "appropriately" for each hanging mass (i.e., at their lowest points in their motion), the conservation of energy yields the *angular velocity* of the pulley at the end of the period as:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

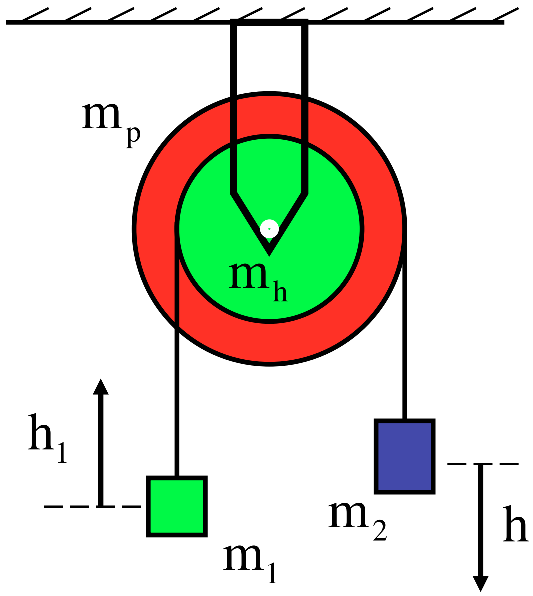
$$0 + \left(m_2 gh\right) + 0 = \left(\frac{1}{2}m_1 v_1{}^2 + \frac{1}{2}m_2 v_2{}^2 + \frac{1}{2}I_{p,h}\omega^2\right) + \left(m_2 g h_1\right)$$

$$\Rightarrow \quad \left(m_2 gh\right) = \left(\frac{1}{2}m_1\left(\frac{2}{3}R\omega\right)^2 + \frac{1}{2}m_2\left(R\omega\right)^2 + \frac{1}{2}\left(\left(\frac{2m_h}{9} + \frac{m_p}{2}\right)R^2\right)\omega^2\right) + \left(m_1 g\left(\frac{2}{3}h\right)\right)$$

$$\Rightarrow \quad \omega = \sqrt{\frac{\left(m_2 gh\right) - \left(\frac{2m_1 gh}{3}\right)}{\frac{4}{18}m_1 R^2 + \frac{1}{2}m_2 R^2 + \left(\frac{m_h}{9} + \frac{m_p}{4}\right)R^2}}$$

g.) What are the velocity magnitudes for both masses for the situation in #e?

$$v_2 = R\omega \text{ and } v_1 = \left(\frac{2}{3}R\right)\omega.$$



h.) What is the hub's *angular momentum* L for the situation in #e?

$$L = I_{hub}\ \omega$$

$$= \left(\frac{1}{2}m\left(\frac{2}{3}R\right)^2\right)\omega.$$